**Mechanics**

**Equations of Equilibrium**

FYI, torques, τ, are called moments, M, in engineering. General statics equations are:



**Net Force and Net Torque from a Distributed Load**

You can calculate the net force and moment of a distributed load. Let’s say d**F**(r) = **f**(r)d3r. The net force is clearly:



Suppose the force is always in a constant direction, though of varying magnitude perhaps, like this: **f**(r) = f(r). Then we’d have:



where the moment **r**cf, which I’ll call the center of force, is:



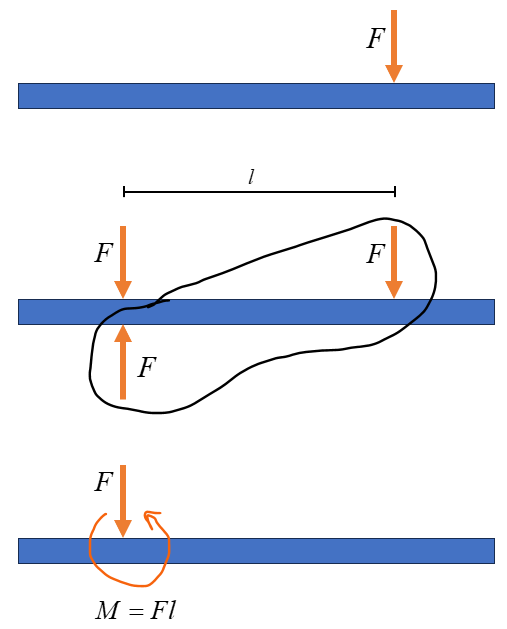
If we specialize to weight, so that f(r) = ρ(r)g, then we’d have:



If the force is non-constant, we can just break it up into its three components, and do this same manipulation for each component.

**Equivalent Force/Moment Combinations**

Can move a force at the expense of a couple. Twould work as follows.

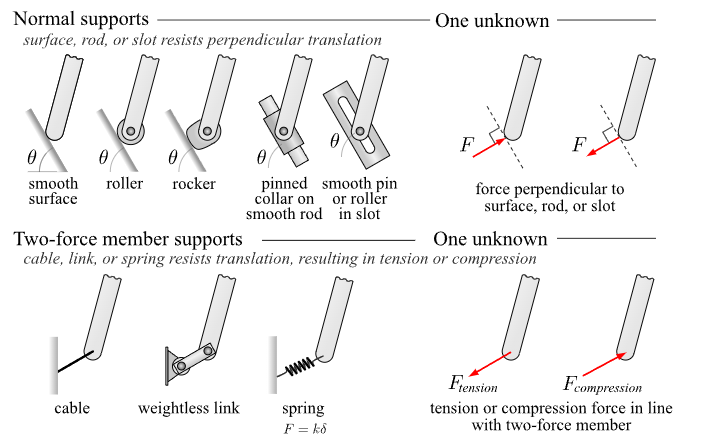


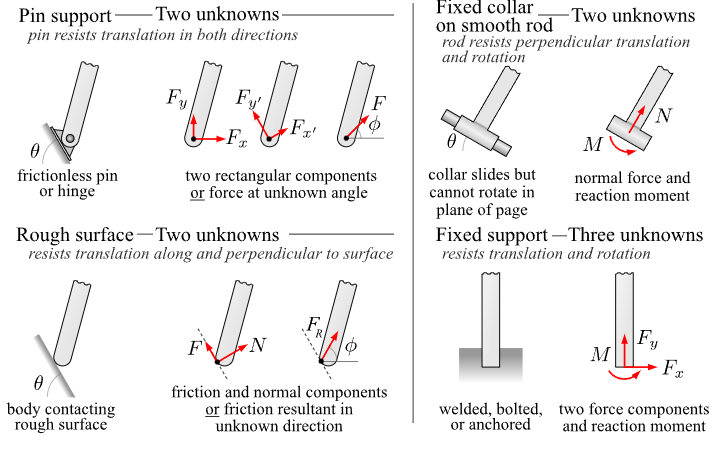
**Compendium of Contact and Distributed Forces**

Forces exerted by different contacts:

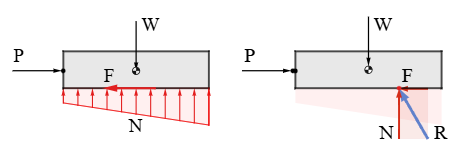
* pin with no friction exerts a longitudinal along the member connected to it.
* pin with friction would exert ?
* roller exerts force perpendicular to the surface it’s rolling on
* arbitrary connection would exert an arbitrary force and torque
* rough surface exerts normal force and planar friction force

Here’s a diagram,





Friction and Normal forces are the two most common I guess. Both are distributed forces, as kind of illustrated below.

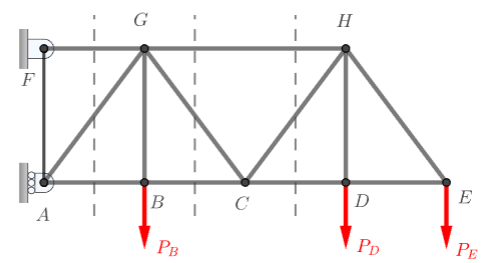


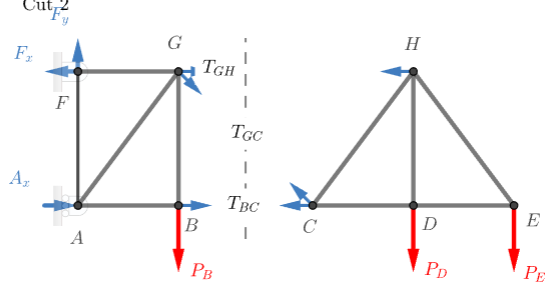
We know that N = ∫n(x)dx = W from N2L in y-direction. But what is n(x)? And we know Fs (called F in diagram) = ∫fs(x)dx = P, but what is fs(x)? Not sure either. But we can know from N2L where the N and Fs can be considered to be acting, by saying Σ**M** = 0. And we can turn this into a single resultant force R.

**Ways to Solve for Forces on Trusses**

How to solve for forces?

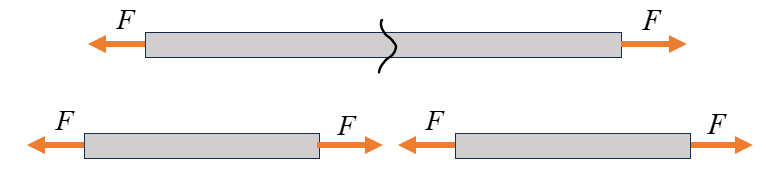
* Method of joints – solve for the forces on all of the joints. Alternatively, could also just solve for the forces on each of the members.
* Method of sections – can cut an object in such a way that we only have one unknown force and/or moment.

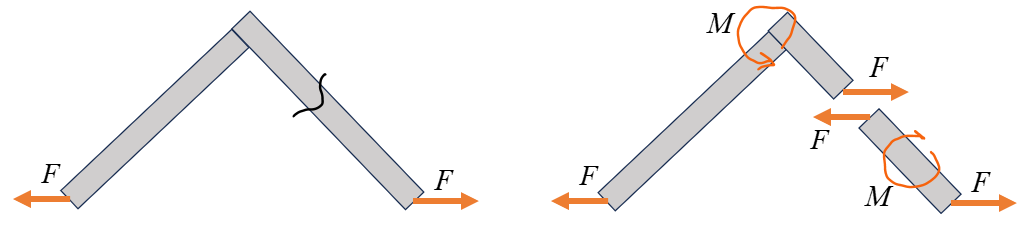




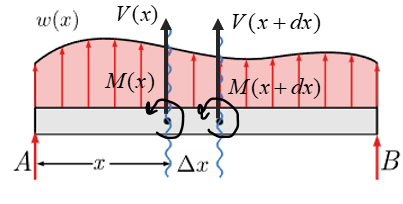
**Internal Forces plus Shear Forces and Bending Moments**

Internal forces look can be resolved as below. Can ascertain by requiring that each piece be in equilibrium separately. Introduction of moments is necessary if the translationally equilibrating force would result in a net moment that has to be gotten rid of. The moment would just be equal and opposite to the moment created by the two F’s.





What about under a load f(x) = w(x)? In that case where will be a force V(x) and moment M(x) that the section of board to the left of x will exert on the section of board to the right of x.



We can work out what these are. Look at the section Δx (or dx). Note that the forces on that segment will be V(x)and -V(x+dx) and the moments will be M(x) and -M(x+dx).



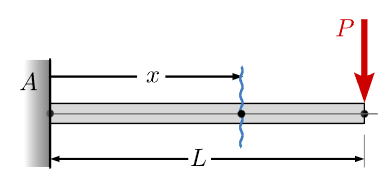
And,



So,



Consider a weightless beam.

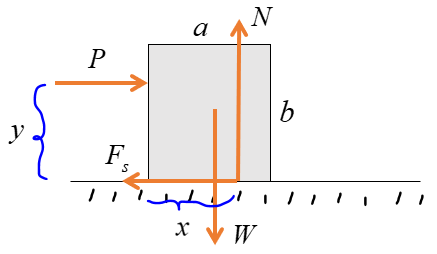


Looks like V(0) would have to be P, for the bar to be in equilibrium. And then since dV/dx = -w(x) = 0, we have V(x) = P everywhere. Also can say M(0) = PL, and since dM/dx = -V(x), we have M(x) = -Px+PL = P(L-x). If the beam were weighted, then we’d have. V(0) = P + W and,



**Slipping and Tipping**

Say we have a box. We’d like to know if it will remain static, slip, or tip, under application of some applied force. We know everything except N, Fs, and x. So with three unknowns we can solve for everything.



Well, first we can see if it will remain static or not. If it remains static (set O at point x) for M equation:



If Fs > μsN, then it will start to slip. If x > a, then it will start to tip. If both, then both I guess.